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Robust Lexicographical Ordering of Images: A Survey

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ABSTRACT: Lexicographical ordering approach correlates the pixel values and reconstructs the correlation to a new color space that is essential for human perception. A vector represents a pixel in color space and vectorial ordering is required for comparison between the vectors. In this paper, a Robust Lexicographical Ordering approach is proposed that overcomes the drawbacks of the existing approaches and arranges the component rank effectively that can be practically applied in various fields. Initially a trance of Robust Lexicographical Ordering is seen and later the results are analyzed in various applications such as noise devaluation and edge detection.
KEYWORDS: Robust Lexicographical Ordering, Vectorial Ordering, Edge Detection, Noise Devaluation

1. INTRODUCTION

Firstly, a technique based on complete lattice theory, set theory, topology and random functions applied to digital images was known as Mathematical Morphology (MM). Object precision and feature indexing that was computationally efficient was detected using this technique. MM was initially used for binary images and later for grayscale images. With the prevailing color images, the ambiguity of ordering of vector pixels constitutes the main obstacle. Among various ordering methods lexicographical ordering is constantly used with the aim of enhancing morphological operators. The major drawback in lexicographical ordering is that the highest priority is given to the first image channel during lexicographical cascade that exploits the inter channel relations. Hence α-modulus lexicographical ordering was introduced that met only the theoretical requirements by providing valid operator's but its practical use was limited.In order to counter this hindrance, Robust Lexicographical Ordering is proposed. The rest of this paper is organized as follows. Section 2 describes the theoretical background of MM operators. In section 3, the various vectorial orderings in color morphology are discussed. Then in section 4, lexicographical ordering and its dimensional representation of images is described. Next in section 5, an abstraction of α- lexicographical ordering is depicted. In section 6, Robust Lexicographical Ordering approach is analyzed. Finally section 7, is devoted for concluding the remarks.

2. MORPHOLOGICAL PROCESSING

The framework of morphology and the issues in MM are discussed in this section. When the complete lattice theory is considered as morphology framework literally, given an image i: ε→R where 'ε' is an arbitrary non-empty set and 'R' is the intensity range of the complete lattice structure which means that 'R' is a non-empty set armed with a partial ordering such that every non-empty subset P⊆R has a greatest lower bound ˄P (infimum) and a least upper bound ˅P (supremum).
Therefore, the set of images I (ε, R) is also a complete lattice.There are two key issues in MM such as Morphological operators and Structuring Element (SE). For a grayscale image, morphological operators are stationed on the concepts of infimum (inf) and supremum (sup). For example, erosion (ε_b) and dilation (δ_b) of a grayscale image 'I' by a flat SE 'b' is represented as follows:

$$\varepsilon_b(i)(x)=\inf_{s\in b}[i(x+s)] \quad (1)$$

$$\delta_b(i)(x)=\sup_{s\in b}[i(x-s)] \quad (2)$$

In grayscale image, a pixel is represented by one scale value. Infimum and supremum can be easily denoted by ranking. On the other hand a color image consists of multi scale images approximately three scale images. Hence one color pixel consists of three scale values that clearly depicts that the method of grayscale MM is not applicable for color MM which means that the ranking of color pixels directly by scale based pixel ordering is not suitable for color image as it is widely used only in grayscale morphology of images. Therefore the morphological operator for color images needs to be defined. In a color image, the color pixel is considered as a vector at each pixel. The vectorial erosion and dilation can be indicated by vectorialextrema operators such as "sup_v" and "inf_v" as follows:

$$\varepsilon_b(i)(x)=\underset{s\in b}{inf_v}[i(x+s)] \quad (3)$$

$$\delta_b(i)(x)=\underset{s\in b}{sup_v}[i(x-s)] \quad (4)$$

Equations (3) and (4) clearly depict that a correct ordering of vectors must be defined to extract the infimum and supremum of the vectors. Hence vectorial ordering is essential in morphology of color images.

3. VECTORIAL ORDERING IN COLOR MORPHOLOGY

The key note to define a well suited ordering is to extend MM to color images.The vector ordering is classified into four groups [1,2,3] as shown in Fig.1.

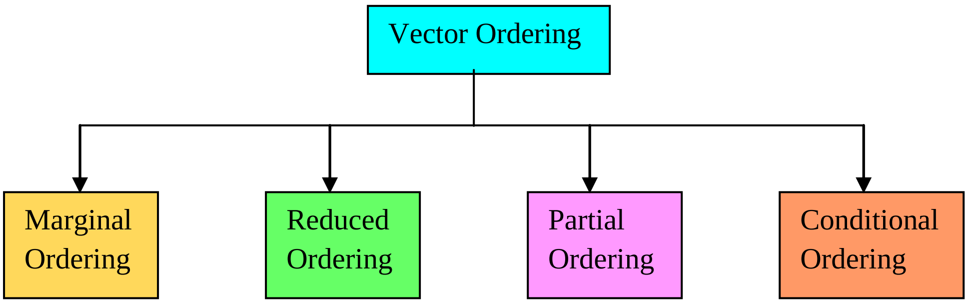


Figure 1:Classification of Vector Ordering

3.1 Marginal Ordering

Marginal ordering method orders the color vector components independently. A color component is partitioned into a set of components and then a uni-variant ordering is applied on each of the component. All the components are then combined to form the output. This method is also known as component wise ordering as data is ordered individually for each of the components. Marginal ordering is thus not a real vectorial ordering approach. Morphological methods for grayscale images can be applied directly using this approach. The major drawbacks of this approach are:

- This approach portrays the color artifacts into the output image since new color vectors are not present in the input image [2].
- The inter component correlation is neglected since each image channel is processed in distinct manner. This leads to the ignorance of the overall information that is used to enhance the quality of results.

3.2 Reduced Ordering

Reduced ordering method analyzes the color vector ordering depending on some scalars that is computed from each of the vector components. The computation corresponds to different measure such as projections or distances. Let us consider an illustration wherein, Reduced Ordering method orders three component vectorial values based on the distance from a reference vector [4].

As a result, the ordering is reduced to a single dimension ordering. This depicts that the output image does not depend only on the input image, but also on the distance measured with respect to the reference vector. The major drawback of this approach is that ambiguity may be seen in the resultant data due to the existence of more than one extreme [5,6].

3.3 Partial Ordering

Partial ordering method depends on the segregation of vectors into groups. These groups are eminent based on rank or extremeness. This can be achieved using sets like convex hull. These sets are more often geometric in nature and is well suited for inter relations between components. The major drawback of this approach is that itleads to multiple extremes [1].

3.4 Conditional Ordering

Conditional ordering method computes the color vector ordering through marginal components that are sequentially chosen depending on different conditions. This ordering method is also known as lexicographical ordering. According to ranking position, the components that are not participating in the process of comparison are listed. Therefore, the vectorial ordering is determined by the set of ranked marginal components. Conditional ordering restricts the ordering process to one or more vector components, while the rest are conditioned on them. Hence, this approach is best suited for areas where few components are more privileged than the rest [1].

4. LEXICOGRAPHICAL ORDERING

Lexicographical Ordering is one variant of Conditional Ordering that is nearly popular. Lexicographical method is the order in which dictionary words are arranged in a sequential manner that is first the order is determined by a component, followed by a second one and finally by the third [6]. Let $p=\{p_1,p_2,p_3\}$ and $q=\{q_1,q_2,q_3\}$ be the two random vectors then lexicographical ordering of these two vectors is represented as follows:

$$p<q \text{ if } \begin{cases} p_1<q_1 \text{ or} \\ p_1=q_1 \text{ and } p_2<q_2 \text{ or} \\ p_1=q_1 \text{ and } p_2=q_2 \text{ and } p_3<q_3 \end{cases} \quad (5)$$

4.1 Representation of 1D matrix vector

Consider an N * N image. The representation of 1D matrix vector is $x(n_1,n_2)$. Fig 2 shows 1D representation as shown below.

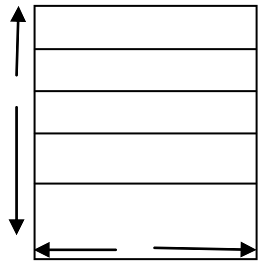


Figure 2: 1D matrix vector representation

Create an N^2 point 1D sequence by integrating the columns by x as shown in Fig 3.

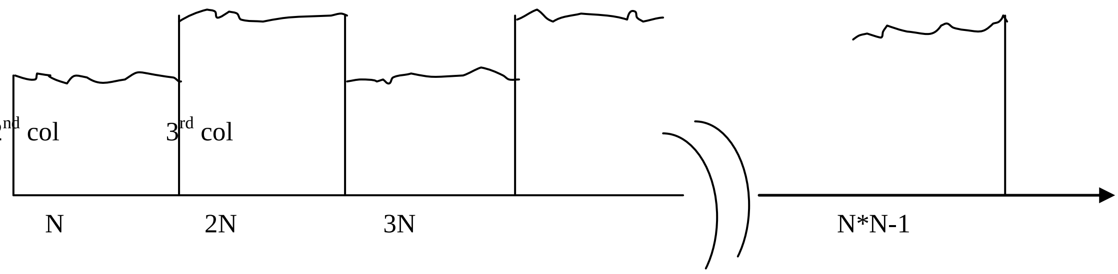


Figure 3: 1D sequential representation

The equation representing the 1D sequence as follows:

$$g\,(Nn_1+n_2) = x(n_1,n_2) \quad (6)$$

where, n1→column index, n2→row index and g→image.

4.2 Representation of 2D matrix vector

Consider a finite sized image g[m,n] using a long vector via lexicographic ordering. If g[m,n] has an M*N domain matrix defined as:

$\{g[m,n]\in R: m=0,\ldots,M-1, n=0,\ldots,N-1\}$(7)

then, the corresponding vector ‘y’ is of length MN and the i^{th} element of ‘y’ is given by the equation as follows:

$$y_i=g[m(i),n(i)],\ i=1,\ldots,MN \quad (8)$$

where vector index ‘i’ maps to pixel coordinates [m(i),n(i)] as follows:

$$m(i) \overset{\Delta}{=} (i-1) \bmod M \quad (9)$$

$$n(i) \triangleq \left\lfloor \frac{i-1}{M} \right\rfloor \quad (10)$$

The index 'i' begins from 1 and indices[m,n] begin with 0. In case of signal processing conventions with regard to consistency, Spatial index [m,n] corresponds to i=1+m+nM which is defined as follows:

$$g[m,n] = y_i \,|_{\, i=1+m+nM} \quad (11)$$

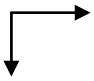
The relationship between vector 'y' and the 2-Dimensional image g[m,n] is represented in Table I.

Table I: Relationship between the vector 'y' and the 2D image

y_1=g[0,0]	y_2=g[1,0]	……………	y_N=g[M-1,0]
y_{M+1}=g[0,1]	y_{M+2}=g[1,1]	……………	Y_{2M}=g[M-1,1]
		: :	
$y_{M(N-1)+1}$=g[0,N-1]	$y_{M(N-1)+2}$=g[1,N-1]	……………	y_{MN}=g[M-1,N-1]

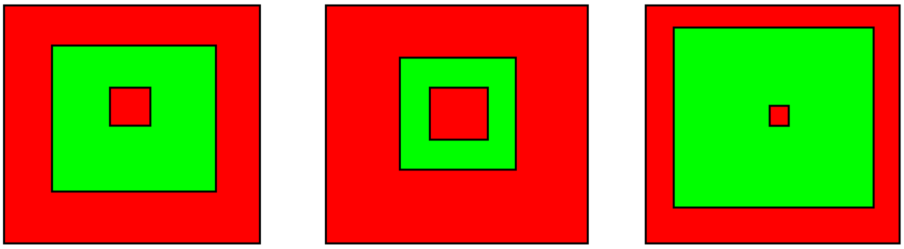
m

n



The lexicographical ordering is widely used due to its theoretical properties since there are no vectors in the processing results and hence prevents the delusion of false colors in color morphology and also the negative effect of pixel signatures during the classification of morphological images.

Lexicographical ordering has a major drawback, where the first component of matched vectors carries the majority of data when compared to the remaining image components that is limited in practice. For example, in RGB color space, the red channel is necessarily prioritized whereas it is sufficient to permute the bands such as GRB to shift the priority to green channel as shown in Fig.4.



(a) **(b)** **(c)**

Figure 4:(a) represents the original image **(b)** Vectorial dilation based on lexicographical ordering (R→G→B) **(c)**Vectorial dilation based on lexicographical ordering (G→R→B) with a 21*21 square SE.

4.3 Vectorial Erosion

Consider an illustration of another image in Fig.5 where vectorial erosion is applied on the original image.Consider a SE of 3*3 square where vectorial erosion is applied as shown in Fig.6. The foreground pixels are represented by 1's and the background pixels by 0's. Each element of the foreground pixels are considered in the input image. For each of the foreground pixel, the SE is superimposed on top of the image such that the origin of SE coincides with the input pixel coordinates. The main purpose of vectorial erosion is to eliminate any foreground pixel that is not completely surrounded by other white pixels. These pixels remain at the edge of white regions and hence the foreground regions shrink [6]. Fig.7 shows the foreground regions that have been reduced after vectorial erosion is applied.

Figure 5: Original image

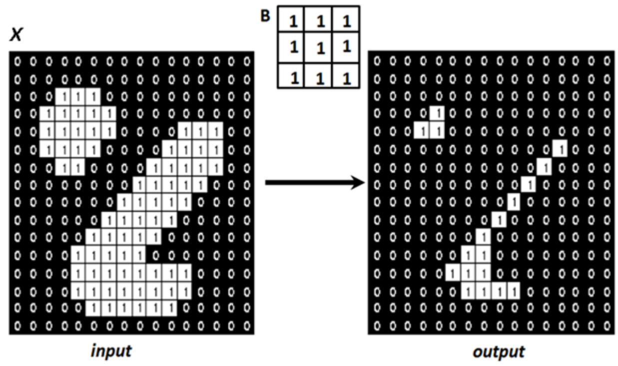


Figure 6: Vectorial Erosion using 3*3 SE

Figure 7: Vectorial Erosion

4.4 Vectorial Dilation

Consider a SE of 3*3 square where vectorial dilation is applied as shown in Fig.8. In order to compute the dilation of an input image, each of the background pixels in the input image is considered. For each background pixel the SE is superimposed on top of the input image such that the origin of SE coincides with the input pixel position. If one pixel in SE coincides with the foreground pixel then input pixel is set to foreground value. If all the respective pixels in the image are background, then the input pixel is left at the background value.

The main purpose of this operation is to set to the foreground color to any of the background pixels that has a neighboring foreground pixel. These pixels lie at the edges of white regions and hence the foreground regions increase. Fig.9 shows the background regions that have been increased after vectorial dilation is applied. Consequently, this scenario, led to an inefficient profiteering of the inter component relations. Therefore, in order to overcome this drawback, α-Lexicographical ordering was proposed.

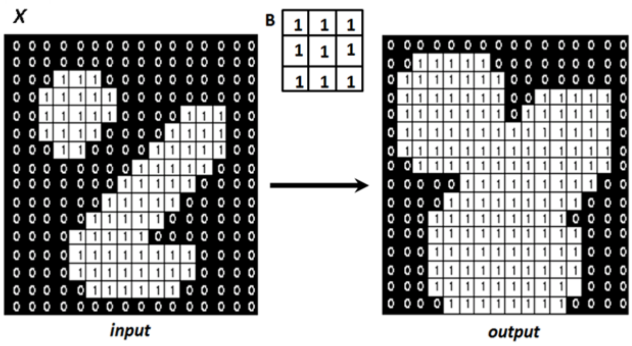


Figure 8: VectorialDilation using 3*3 SE

Figure 9: VectorialDilation

5. □-LEXICOGRAPHICAL ORDERING

The first pursuit, aiming to decrease the priority of the first vector component during lexicographical comparison was proposed by Ortiz [7], and this proposed method was known as α-Lexicographical ordering.

$$\forall v, v' \in R^n, v < v' \Leftrightarrow \begin{cases} v_1 + \alpha < v_1', \text{ or} \\ v_1 + \alpha \geq v_1' \text{ and } [v_2, \ldots, v_n]^T <_L [v_2', \ldots, v_n']^T \end{cases} \quad (12)$$

where, $\alpha \in R^+$. At the end of increasing the equivalence occurrence within the first vector dimension, the α argument is used, as such a scalar value $v_1$becomes equivalent to all values in the interval $[v_1-\alpha, v_1-+\alpha]$, thus allowing comparisons to reach the second dimension.Further, α-Lexicographical ordering approach was enhanced by Aptoula and Levefre[8], by another approach known as α-Trimmed Lexicographical Extrema. This approach is an iterative approach depending on the principle of α-Trimming where given a set ‘V’ containing ‘k’ vectors with respect to maximum case, initiating from the first dimension, the contents of the set ‘V’ are sorted based on this dimension and$\lceil \alpha \times k \rceil (\alpha \in [0, 1])$, the greatest vectors that are sorted are considered as the new set ‘V’. On repeating this process at each stage for each of the dimension, the initial vector set is thereby reduced gradually to the desired extremum.
The major drawback is that this approach does not support binary relation of vectors but depends on cumulative extremum computation where ordering does not exist and hence leads to pseudo morphological operators.

A theoretical approach was then proposed by Angulo and Serra[9] and was known as α-modulus lexicographical ordering.

$$\forall v, v' \in Z^n, v < v' \Leftrightarrow [\lceil v_1/\alpha \rceil, v_2, \ldots, v_n]^T <_L [\lceil v_1'/\alpha \rceil, v_2', \ldots, v_n']^T \quad (13)$$

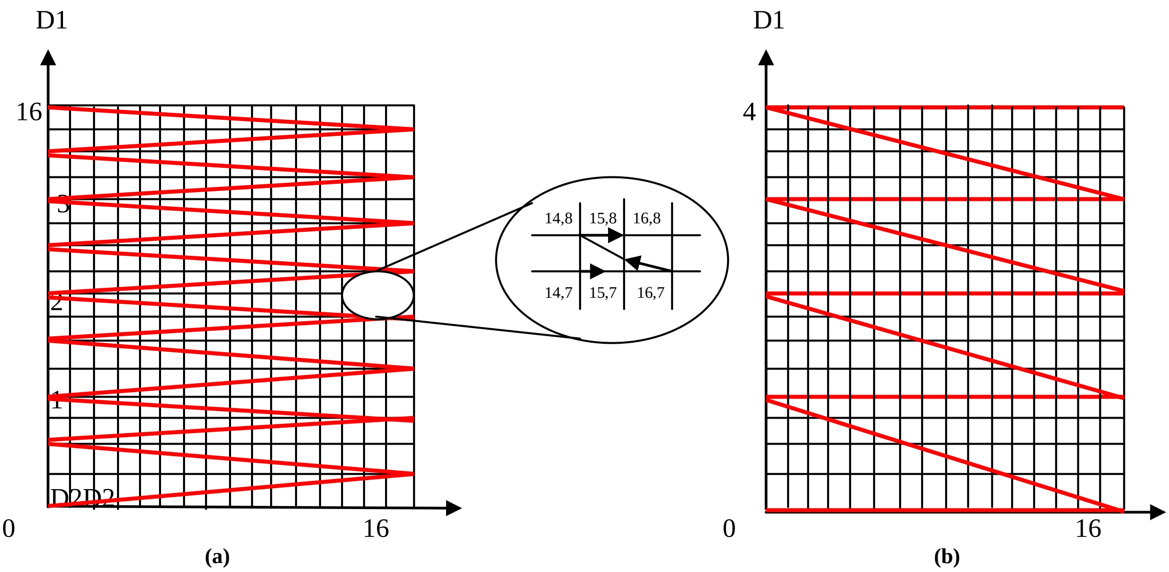


Figure 10: Space Filling Curves (SFC) in bi- dimensional space. (a) represents Lexicographical Ordering (b) represents α-modulus Lexicographical Ordering (α=4)

The arrows in Fig.10 denote the increasing vectorial co-ordinates direction. This main objective of this method is to form equivalence groups within the first dimension. It reckons on a quantization through division by a constant 'α' succeeded by a rounding off that curtails the dynamic margin of the first dimension thereby allowing a larger number of comparisons to reach the second.

This results in a valid morphological operator. From Fig.10, it is clear that SFC maps all the points of multi dimensional space onto anuni-dimensional space where vectors are ordered according to co-ordinate positions on SFC [10]. Since Lexicographical Ordering corresponds to a bijection, its SFC will pass once from all points of a bi-dimensional distinct space D1xD2 as shown in Fig.10(a) where high priority attributed to D1 depicts high frequency by horizontal curves.

Fig.10(b) depicts quantized form D1 of dimension D1 with α=4 that leads to creation of equivalence groups. This method discards the anti-symmetry property of lexicographical ordering. Also the equivalence between vectors does not occur within the quantized space.

$$\forall v, v' \in Z^n, v = v' \Leftrightarrow (v \leq v') \wedge (v \geq v') \quad (14)$$

They occur between vectors using their original un-quantized values:

$$\forall v, v' \in Z^n, v = v' \Leftrightarrow [v_1, v_2, \ldots, v_n]^T = [v_1', v_2', \ldots, v_n']^T (15)$$

Equation 13 satisfies only theoretical requirements of ordering and hence results in valid morphological operators. The practical use of α-modulus lexicographical ordering is limited and reckons on an implicit assumption on the dimension to be quantized.

6. QUANTIZED □-LEXICOGRAPHICAL ORDERING

The given distinct pixel range interval [0,255] is realized as a quantization algorithm by associating to each value, a group of equivalence whose size is computed by a user defined function 'f' and is limited by a value of $\alpha \in N^+$. Hence Equation 13 is reformulated as:

$$\forall v, v' \in Z^n, v < v' \Leftrightarrow [w_1, v_2, \ldots, v_n]^T <_L [w_1', v_2', \ldots, v_n']^T (16)$$

where, w_1 and w_1' depict equivalence group of v_1 and v_1' obtained through an Algorithm. In complex inter-channel relation this approach requires repeated use in order to be modeled effectively [11,12,13].

Algorithm: Quantized discrete dimension computation based on erratic priority distribution

Input: I=[p,….,q] ⊆ Z, an array that contains distinct dimension to have its dynamic margin reduced.
α∈N*, a parameter that sets the maximum size of equivalence groups within I.
f: N → [0,1], a function modeling the desired priority distribution within I.
Output:J ⊆ N, the array consisting of new quantized dimension.

```
tmp ← 0
for i ← p to q do
k[ p x f (←I[ i – p ] ) ]
for j i to i + k do←
        J [ j - p ]← tmp
end for
tmptmp + 1←
    i = i + k
end for
```

7. ROBUST LEXICOGRAPHICAL ORDERING

In lexicographical ordering, the result of the output image is decisive by the first component and least by the last component. In RGB color space, the maximum value component out of the three components determines the image nature and hence this component is selected as first component. Likewise, the minimum value component is considered as last component. Hence in this approach, prior to ordering arrangement, the maximum, minimum and median of three components is obtained.

Let us consider a color image ' f ' such that f: $R^2 \rightarrow RGB$ and flat SE 'b'. Each of the components of RGB is computed as follows:

com_R = ∑ f_R (p) ∑f_R (p)

βp∈R+ (1-β) p∈b (17)

R b

com_G= ∑ f_G (p) ∑f_G (p)

βp∈R+ (1-β) p∈b(18)

R b

com_B= ∑ f_B (p) ∑f_B (p)

βp∈R+ (1-β) p∈b(19)

R b

where, $f_R(p)$, $f_G(p)$, $f_B(p)$ denote the red, green and blue component respectively, com_R, com_G and $com_B$are ranked to depict the maximum, median and minimum. Maximum is arranged as the first component and minimum as the last component. Then depending on Equation.(5), Robust Lexicographical Ordering is achieved. β ($0 \leq \beta \leq 1$) is a weighted coefficient that has an impact on the visual effects of the output image.
If β is too small then the output image has a poor visual effect.

Figure 11: Original Image

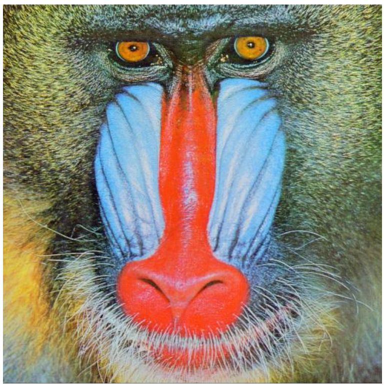
Figure 12: Erosion by Robust Lexicographical Ordering approach

Figure 13: Dilation by Robust Lexicographical Ordering approach

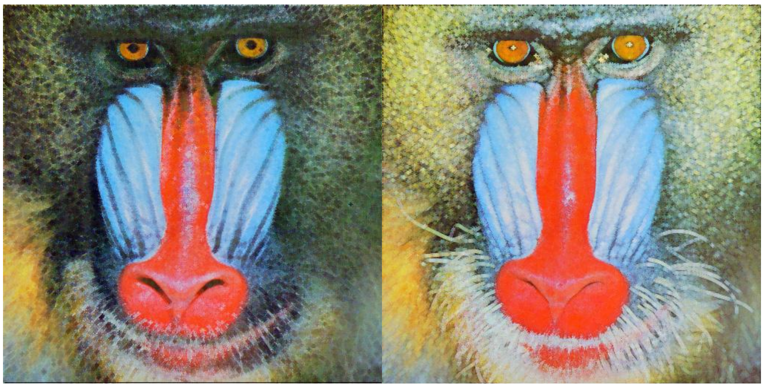
Fig.12 and Fig.13 use Fig.11as original image that is enforced by vectorial erosion and dilation with 7*7 flat SE and $\beta = 0.8$.Depending on erosion and dilation two other morphological operators such as opening and closingare defined as shown in Fig.14. An opening is erosionsucceeded by dilation and closing is a dilation succeeded by erosion as follows:

$$\gamma_b (f) = \delta_b [\varepsilon_b (f)] \tag{20}$$

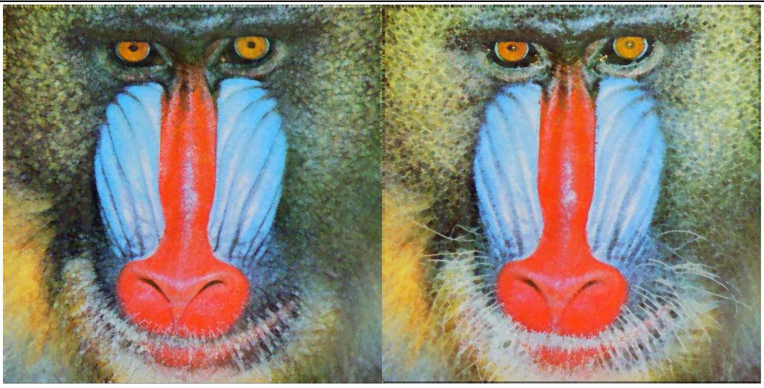
$$\phi_b (f) = \varepsilon_b[\delta_b (f)] \tag{21}$$



(a) Original Image



(b) Erosion (c)Dilation



(d) Opening **(e)** Closing

Figure 14: Example of Robust Lexicographical Ordering approach

7.1 Applications

The applications of Robust Lexicographical Ordering approach such as noise devaluation, edge disclosure are discussed in this section.

7.1.1 Noise Devaluation

The Normalized Mean Square Error (NMSE) is computed as quantitative measure as follows:

$$NMSE = \frac{\sum_{p=1}^{N} \sum_{q=1}^{M} \| f(p,q) - f'(p,q) \|^2}{\sum_{p=1}^{N} \sum_{q=1}^{M} \| f(p,q) \|^2} \tag{22}$$

where 'N' and 'M' represent dimensions of the image while f (p, q) and f ′ (p, q) denote the vectorial pixels at position (p, q) for the original, filtered and noisy images, while || . || depicts Euclidean norm. To devaluate the noise by MM a method known as OCCO is defined pixel wise average of Open Close and Close Open as follows:

$$OCCO_b(f) = \tfrac{1}{2}\gamma_b[\phi_b(f)] + \tfrac{1}{2}\phi_b[\gamma_b(f)] \tag{23}$$

where 'ϕ' and ' γ' denote operators of vectorial closing and opening. The images have been filtered using a square shaped SE of size 3*3 pixels.

Fig.15(a) clearly denotes the host image that with Noise that is used for test analysis and Fig.15(b) depicts the host image after removal of noise. These figures clearly represent the exaggerated image of the parachute where Robust Lexicographical Ordering approach can be applied.

(a) (b)

Figure 15: (a) Represents Host Image with Noise, (b) Represents Host Image after removal of Noise

7.1.2 Edge Detection

The morphological gradient for a grayscale image ‘f’ is denoted as [14]:

$$\nabla f = \delta_b(f) - \varepsilon_b(f) \tag{24}$$

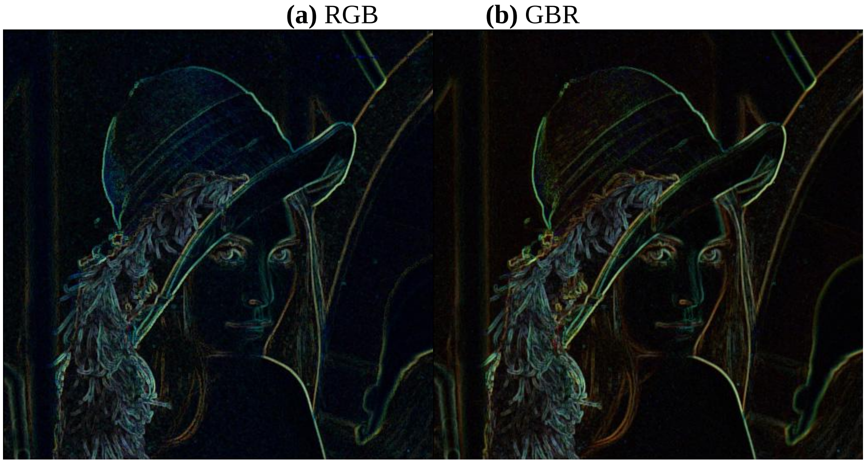
where, ‘f’ denotes the color image, $\delta_b(f)$ and $\varepsilon_b(f)$ are color vectors. There exist several color morphological gradients and hence the formula is [15]:

$$\nabla f = \| \delta_b(f) - \varepsilon_b(f) \| \tag{25}$$

where, || || denotes the Euclidean Norm.

Figure 16: Host Image

(a) RGB **(b)** GBR



(c) BRG **(d)**Robust Lexicographical Ordering approach

Figure 17: Edge image of Lexicographical Ordering approach

Fig.16 is the host image for testing. Fig.17 represents the edge images of lexicographical ordering approach. Fig.17(a), Fig.17(b) and Fig.17(c) represents the RGB, GBR and BRG edge image of the host image as shown in Fig.16. The RGB edge image has more red edges, GBR has more green edges and BRG has more blue edges. Since color of host image is inclined to red the edge image seen in Fig.17(d) and the RGB image seen in Fig.17(a) appears to be similar. In Fig.17(d) the coefficient of β=0.6. After deep test analysis it is clear that the edges makes little or no difference with varying β values.

8. CONCLUSION

Lexicographical ordering has proved to be an invaluable vector ordering methodology for MM as it possesses multiple desirable theoretical properties and makes it possible to customize the order of comparison among image channels. This paper discusses the various orderings with their advantages and drawbacks. Also α-Lexicographical Ordering, Quantized based approaches as well as Robust Lexicographical Ordering approach along with its applications is briefly discussed.

9. FUTURE ENHANCEMENT

The future enhancement includes more application oriented use that serves as a boon for the field of image processing and also requires immediate attention with the increasing digital image technology with numerous graphical editors and tools.

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